11th WCRR World Congress on Railway Research 2016
29th May – 2nd June
Milan - Italy

Cost-Based Optimization of Track Geometry Inspection

Iman Soleimanmeigouni[1*], Alireza Ahmadi[1], Christophe Letot[2], Arne Nissen [3], Uday Kumar[1]

[1] Division of Operation and maintenance, Luleå University of Technology, Luleå, Sweden

[2] Machine Design and Production Engineering Lab, Research Institute for the Science and Management of Risks, University of Mons, Mons, Belgium

[3] Trafikverket, Luleå, Sweden

Contact: iman.meigouni@ltu.se

Abstract

Track geometry bear huge static and dynamic forces that accelerate degradation process. As a result, railway track should be inspected regularly to detect geometry faults and to plan maintenance actions in advanced. An inspection plan that minimizes track maintenance cost is highly desirable by infrastructure managers. This paper proposes constructing an integrated model to identify the optimum track geometry inspection interval. To this end, it develops a long term prediction model combining degradation, shock event, and tamping recovery models. It applies the Wiener process to model track geometry degradation, simulates shock event times using an exponential distribution, and uses a probabilistic model to model recovery after tamping. With the proposed integrated model and simulation, it is possible to identify the optimum track geometry inspection frequencies that minimize total track maintenance costs.

Keywords: Degradation, inspection, Track geometry, maintenance cost, optimization, recovery model, shock model, simulation, tamping, Wiener process

1. Introduction

Railway track degrades with age and usage, losing its functionality over time. Inspection and maintenance tasks are intended to detect geometry faults, to control the degradation process, and to protect against unacceptable consequences. In order to define an effective inspection interval, constructing a long term prediction model for track geometry condition is essential. To create such a model, degradation, shock event, and recovery models can be constructed and combined. Since inspection has a profound impact on total cost, it is necessary to identify the cost effective inspection interval as one of several alternatives to minimize total track maintenance cost.

This paper proposes an integrated model to identify the cost effective track geometry inspection interval. To model track geometry degradation, it suggests applying the Wiener process to each track section. In addition, it simulates shock event times using an exponential distribution. Finally, to model tamping recovery values, it takes a probabilistic approach by considering the effect of track geometry condition before tamping on the recovery value of tamping. The parameters of the respective models are estimated using data collected by the measurement wagon STRIX/IMV200. The paper develops a cost model to compute the total cost of the inspection strategy, considering the costs associated with tamping intervention, inspection, and penalties, and tests the model using a case study from the Swedish railway system.

The rest of this paper is organized as follows. Information on the railway line used as a case study is provided in Section 2. Section 3 describes the application of the Wiener process for track geometry degradation modeling. The proposed approach to modeling the shock events is explained in Section 4, and the suggested recovery after tamping model is presented in Section 5. Integration of the degradation, shock event, and recovery models for long term prediction of the track geometry condition of each track section is explained in Section 6. The total costs of various inspection scenarios are compared in Section 7. Finally, Section 8 provides a conclusion.

2. Line information and data collection

The Main Western Line in Sweden (Västra Stambanan) is the main railway line between Stockholm and Gothenburg. The maximum speed of trains on the Main Western Line is around 200 km/h. Line 414 between Järna and Katrineholm Central Station is used as the case study. The relevant data on the line were collected from 2007 to 2015 by Optram, a system used since 2007 by Banverket (the former Swedish Rail Administration) and Trafikverket (Swedish Transport Administration) to study measurements of the track and overhead lines [1]. Numerous environmental and structural factors affect track geometry condition, and various different geometry degradation behaviors can be observed along the track length. To acquire a better understanding of track geometry condition over the track length, line section 414 was divided into 200-meter track sections. The study uses the measurement data gathered by measurement wagon STRIX/IMV200.

3. Degradation model

Modeling track geometry degradation can take either a stochastic or a deterministic approach. Readers are referred to Soleimanmeigouni et al. [2] for further study on different track geometry degradation models. To accurately model track geometry degradation, uncertainty must be considered in the degradation modeling process. The present study uses the Wiener process to model track geometry degradation because of its ability to capture uncertainty.

The Wiener process has been widely applied to degradation modeling in various fields, e.g. bearings, laser generators and milling machines [3]. A stationary Wiener process is particularly well suited to model the evolution of a degradation mechanism characterized by a linear increase over time with random noise. Such a process has continuous sample paths and independent, stationary and normally distributed increments [4]. A Wiener process-based model is characterized by a drift parameter representing the expected degradation rate and a Brownian motion term to consider the non-monotonic behavior. According to Kahle et al. [5], the formula for the Wiener process can be expressed as:

$$Z(t) = z_0 + \mu t + \sigma W(t) \tag{1}$$

where $Z(t)$ is the degradation measure described by the model, z_0 is the initial degradation, μ is the drift coefficient, σ is the diffusion coefficient, and $W(t)$ is the standard Brownian motion representing the stochastic dynamics of the degradation process [4]. Time in the model can be represented as:

$$t = t_{step\ length} \times n \tag{2}$$

where $t_{step\ length}$ is the time step length and n is the number of steps.

The Wiener process is associated with the concept of Brownian motion. Hence, it can be represented by the following formulation:

$$Z(t) = z_0 + N\left(\mu t, \sigma\sqrt{t}\right) \tag{3}$$

The expected value of the Wiener process with the drift coefficient μ is $E\left(Z(t)\right) = \mu t,$ and the variance is $Var(Z(t)) = \sigma^2 t.$

Using the measurement data obtained by measurement wagon STRIX/IMV200, the drift coefficient and diffusion coefficients are determined using the method of maximum likelihood estimation (MLE) for each track section. For the Wiener process time step, a period of two weeks is selected to simulate the degradation behavior of each track section. In this study, the standard deviation of longitudinal level (SDL) is considered to represent track condition, as vertical defects grow slightly more rapidly than horizontal ones [6]. It should be noted that, it is assumed that traffic will be constant during the time horizon of the study. In order to consider the effect of the change in traffic in degradation model, the track geometry degradation should be modeled as a function of passing tonnage instead of time.

4. Shock model

Track geometry can be expressed by a degradation relevant stochastic model, in which the system fails whenever its degradation level exceeds a specific threshold. In addition to the normal process of degradation, the condition of the track geometry may be subject to shock events causing sudden increments in degradation model parameters [7]. This study assumes shock events are independent and identically distributed (i.i.d). It also assumes the occurrence rate of shock events remains constant and only one shock may occur for a track section in the time horizon of the study. Therefore, there is no need to consider inter arrival time between shock events. An exponential distribution is therefore applied to estimate the time to shock events and expressed as:

$$f(t_s) = \theta e^{-\theta t_s}\,,\;\; t \in [0, \infty) \tag{4}$$

where θ is the occurrence rate of shock events, and t_s is the time of the shock event. To estimate the θ, the MLE method is applied. It should be noted that for a significant number of sections a sudden increment in degradation level between two inspection intervals are observed. Thus estimating the occurrence rate of shock events is possible. For several track sections, there is no shock event in the time horizon of this study; therefore, right censored data are involved in the estimation process. For a right censored section, the only known fact is that the t_s is greater than the time horizon of the study. Hence, the maximum likelihood function for unknown parameter θ is as follows:

$$L(\theta) = \prod_{i=1}^{r} f({t_s}^{i}) \prod_{j=r+1}^{r+m} 1 - F({t_s}^{i}) \tag{5}$$

$$L(\theta) = \prod_{i=1}^{r} \theta e^{-\theta {t_s}^{i}} \prod_{j=r+1}^{r+m} e^{-\theta {t_s}^{i}} \tag{6}$$

where $L(\theta)$ is the likelihood function, $f({t_s}^{i})$ is the probability density function (pdf) of t_s for section number i, $F({t_s}^{i})$ is the cumulative distribution function (cdf) of t_s for section number i, r is the number of sections without censored data, and m is the number of sections with censored data. To find the MLE estimation, the likelihood function should be maximized with respect to the unknown parameter θ. For the case study, using the collected data, we estimate the occurrence rate of shock events $\hat{\theta}$ as 0.0414 per year. Therefore, we use the exponential distribution with the above-mentioned rate to simulate the time to shock event for each track section. Estimating the values of sudden increments in degradation level is the next step. As it mentioned before the shocks are occurred for a number of track sections and enough data is available for finding the distribution of the shock event values. In this regard, different distributions are fitted on the shock event values. The Anderson-Darling test is used as the goodness-of-fit criterion. As shown in Table 1, the lognormal distribution has the smallest Anderson-Darling values. Since for three-parameter lognormal distribution there is no established method for calculating the p-value, firstly the corresponding p-value for lognormal distribution should be examined and then the likelihood ratio test (LRT) p-value for three-parameter lognormal distribution should be examined. Since the LRT p-value for three-parameter lognormal is small we can conclude that three parameter lognormal is significantly better than two parameter lognormal. The three parameter lognormal distribution for value of sudden increment has the following pdf:

$$\mathrm{f}(\mathrm{V}_s) = \frac{1}{(\mathrm{v}_s - \tau)\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\ln(\mathrm{v}_s - \tau) - \mu)^2}{2\sigma^2}\right\} \tag{7}$$

where v_s is the value of a sudden increment in degradation level, μ is the location parameter, σ is the scale parameter,and τ is the threshold parameter . For the fitted distribution, the estimated location, scale, and threshold parameters are -1.8, 0.53, and 0.07 respectively. Therefore, the three-parameter lognormal distribution is used to simulate random values for sudden changes in degradation level.

Table 1. Results for the distribution fitted to shock values

	Anderson-Darling	P-value
Normal	3.34	<0.005
Gamma	1.2	<0.005
Weibull	2.73	<0.01
Exponential	19.71	<0.003
Lognormal	0.608	0.11
3 parameter lognormal	0.412	*

5. Tamping recovery model

Tamping is one of the major activities carried out by infrastructure maintenance managers to remedy track geometry failures. Although tamping will improve the track geometry condition, it cannot restore the geometry condition to an as-good-as-new state. This study assumes the recovery after tamping for longitudinal level is a random variable with a specific distribution. Therefore, it requires a distribution that can be fitted on the recovery values. After the distribution is selected, the parameters of the distribution should be considered as a function of the track geometry condition before tamping. The Anderson-Darling test serves as the goodness-of-fit criterion. As shown in Table 2, the three-parameter Weibull distribution has the smallest Anderson-Darling values and a greater P-value than 0.05. Therefore, the three-parameter Weibull distribution is the best choice to represent the data. The three-parameter Weibull distribution for the recovery value of the longitudinal level has the following pdf:

$$f(R) = \frac{\beta}{\alpha}\left(\frac{R-\gamma}{\alpha}\right)^{\beta-1} e^{\left(-\left(\frac{R-\gamma}{\alpha}\right)^{\beta}\right)} \quad (8)$$

where R is the recovery value for the SDL, β is the shape parameter, α is the scale parameter, and γ is the location parameter. The mean and variance of the distribution can be represented as follows:

$$Mean(R) = \gamma + \alpha\Gamma(1 + \frac{1}{\beta}) \quad (9)$$

$$Var(R) = \alpha^2 \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2 \right\} \quad (10)$$

where Γ represents the gamma function. For the fitted distribution, the estimated location, shape and scale parameters are -0.029, 2.566 and 0.603, respectively. In this regard, to model the recovery after tamping for the longitudinal level and cope with the uncertainty of the recovery after tamping for different track geometry conditions before tamping, the study assumes the recovery after tamping follows the three-parameter Weibull distribution, and the parameters of the distribution are a function of the track geometry condition before the tamping intervention.

Table 2. Results for the distribution fitted to recovery values after tamping for SDL

	Anderson-Darling	P-value
Normal	0.414	0.332
2-Parameter Weibull	0.245	>0.250
2-Parameter Gamma	1.413	<0.005
Logistic	0.689	0.043
3-Parameter Weibull	0.177	>0.500

As the mean and variance of the recovery of the longitudinal level are directly proportional to the scale and location parameters and inversely proportional to the shape parameter, the scale, shape and location parameters are assumed to be a function of the SDL before tamping and are expressed as:

$$\alpha = a\rho + b \quad (11)$$

$$\beta = \frac{1}{c\rho + d} \quad (12)$$

$$\gamma = e\rho + f \quad (13)$$

where ρ is the SDL before tamping, and a, b, c, d, e, and f are the model coefficients. The method of maximum likelihood estimation (MLE) is applied to estimate the value of these coefficients. The estimated parameters are $\hat{a} = 0.896, \hat{b} = 0.091, \hat{c} = 0, \hat{d} = 0.088, \hat{e} = -0.28, \hat{f} = -0.22$.

6. Long term prediction model

By combining degradation, shock event, and recovery models, we can predict the long-term behavior of the track geometry for each track section. By considering different thresholds, this prediction can be used to compare different inspection strategies and select the one that will give the lowest cost. Two main threshold levels are considered in this study. When degradation reaches the first threshold, τ_1, corrective maintenance should be performed on the track section and a penalty cost is incurred. Once degradation passes the second threshold, τ_2, a larger penalty cost is incurred. In this study, tamping interventions are considered corrective maintenance actions. To estimate the time for the degradation level to reach the first and second thresholds, the Wiener process and shock model should be applied simultaneously. By involving the recovery model in the prediction model and by considering equations 14,15,16,17, expressed below, the long term behavior of track geometry condition of each track section can be simulated.

$$x\left(t_1 = t_{step\ length} \times 1\right) = z\left(t_1 = t_{step\ length} \times 1\right) \tag{14}$$

$$\forall\ 1 < n \leq N$$
$$x\left(t_{n+1} = t_{step\ length} \times (n+1)\right) = x\left(t_n = t_{step\ length} \times n\right) + \mu t_{step\ length} + \sigma W(t_{step\ length}) + v_{s_i} - R \tag{15}$$

$$s.t:$$

$$v_{s_i} = \begin{cases} If\ t_{s_i} > t_n \quad and \quad t_{s_i} < t_{n+1} & is\ a\ simulated\ value\ from\ the\ model\ in\ section\ 4 \\ Otherwise & 0 \end{cases} \tag{16}$$

$$\forall\ n = \frac{T}{t_{step\ length}} \quad and \quad n \leq N$$
$$R = \begin{cases} If\ x(t_n) > \tau_1 & is\ a\ simulated\ value\ from\ the\ model\ in\ section\ 5 \\ If\ x(t_n) < \tau_1 & 0 \end{cases} \tag{17}$$

where $x(t)$ is the standard deviation of longitudinal level at time t, n is the step number, N is the maximum number of time steps, $t_n$is time at step number n, t_{s_i} is the time of the i^{th} shock, and v_{s_i} is the value of the sudden increment of the i^{th} shock. In addition, μ , σ, and $W(t_{step\ length})$ are the defined parameters as per equation 1. Figure 1 shows the simulated long term behavior of an arbitrary selected track section from line 414. Note that the simulation was performed for all 111 track sections in the case study.

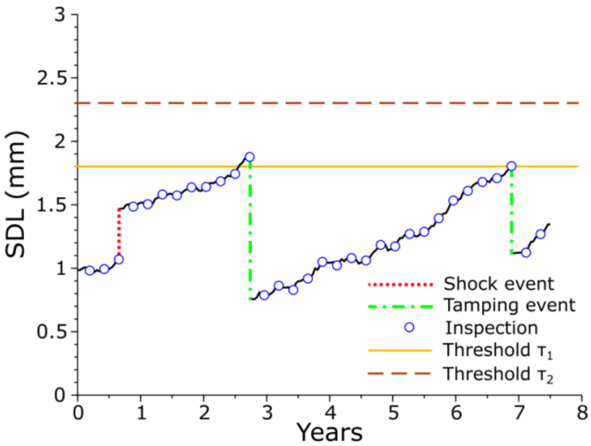

Figure 1. Simulation of long term behavior of track geometry for a 200 meter track section

7. Evaluation of the most cost effective inspection interval

This section compares four scenarios to find the inspection strategy that result in the lowest cost. The inspection intervals for scenario one, two, three, and four are 6, 12, 24, and 48 weeks, respectively. It is assumed τ_1 and τ_2 are 1.8 mm and 2.3 mm respectively for SDL. To find the best inspection scenario among all available choices, the cost function must be defined. The tamping cost, inspection cost, penalty cost for exceeding τ_1, and penalty cost for exceeding τ_2 are all considered in the cost function. The cost function, therefore, is defined as:

$$Expected\ total\ cost = (N_I . C_I) + (E(\mathrm{N_T}) . \mathrm{C_T}) + (E(T_{\tau_1}). C_{\tau_1}) + (E(T_{\tau_2}). C_{\tau_2}) \tag{18}$$

where N_I is number of inspections, $\mathrm{N_T}$ is number of tamping interventions, T_{τ_1} is total duration over τ_1 (days), T_{τ_2} is total duration over τ_2, C_I is inspection cost per 200 meters × total number of sections, $\mathrm{C_T}$ is tamping intervention cost per 200 meters, C_{τ_1} is penalty cost for exceeding τ_1 per day, C_{τ_2} is penalty cost for exceeding τ_2 per day. Based on the study by Arastehkhouy et al. [8], the costs of inspection and corrective tamping are 240 SEK and 5000 SEK respectively for each 200-meter section. The study assumes the penalty costs for exceeding IL and IAL are 200 SEK and 15,000 SEK per day, respectively. The time horizon of this study is assumed to be 7.5 years.

Table 3 shows the simulation results for the four inspection scenarios for a time horizon of 7.5 years.

Table 3. Simulation results of four inspection scenarios

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Expected total time duration over τ_1 (days)	540	960	1,576	2,798
Expected total time duration over τ_2 (days)	0	0	10	90
Expected number of tamping interventions in time horizon	128	125	117	113
Number of inspections in time horizon	64	32	16	8

As it can be seen in Table 3, when 64 inspection is performed on the track, 128 tamping interventions are conducted. It does not mean that in each inspection cycle two tamping is conducted. In fact, it means that 128 tamping interventions are performed on individual track sections on the time horizon of the study. When more inspections are performed on the track, for some sections more tamping would be conducted to prevent exceeding the τ_2 threshold. In fact, since by a shorter interval the tamping may conducted in a lower degradation level, the probability of exceeding the τ_2 threshold will decrease. Table 4 shows the total cost incurred by different inspection scenarios. Inspecting the track every 24 weeks clearly results in the lowest total cost. In scenario 1, the inspection interval is too small and leads to high inspection and tamping costs. When the inspection interval is increased from 12 weeks to 24 weeks, the total time duration over τ_2 is also increased, thereby incurring a significant penalty cost. However, because there are fewer inspections and tamping interventions (see Table 4), the total cost of the third scenario is lower. When the inspection interval is increased to 48 weeks the total time duration over τ_1 and τ_2 increases dramatically, generating a huge penalty cost. Hence, inspection scenario 4 is not cost ineffective.

Table 4. Total cost (SEK) of inspection scenarios $C_{\tau_1} = 200 \; and \; C_{\tau_2} = 15000$

Inspection scenario	6 weeks	12 weeks	24 weeks	48 weeks
Total cost	3,077,600	2,286,800	2,057,600	3,250,800

To identify how changes in C_{τ_1} and C_{τ_2} affect the total cost, a sensitivity analysis is performed; the results appear in Table 5. The total cost of the four inspection scenario is computed for four different combination of C_{τ_1} and C_{τ_2} .Following table 5, by increase the C_{τ_1} to 1000 SEK performing inspection at every 12 weeks will yields the lowest total cost. When C_{τ_1} is high, it is desirable to reduce the total time duration over τ_1 by performing inspections at shorter intervals. A reduction in C_{τ_2} to 5,000 SEK reduces the share of total time duration over τ_2 in the total cost. In this case, as the table shows, the total cost of inspection scenario 4 is significantly low compare to other three combination of C_{τ_1} and C_{τ_2}. Finally, an increase in C_{τ_2} to 50,000 SEK may incur a huge penalty cost by exceeding τ_2. Therefore, the fourth scenario has a significantly higher total cost than other alternatives. In this situation, performing inspections every 12 weeks result in the lowest total cost. In order to have a more comprehensive sensitivity analysis, the effect of changes in model inputs such as parameters of degradation, shock, and recovery models on total cost also can be considered. A change in these parameters may cause a different track geometry behavior. As a result, it will affect the parameters of cost model. However, this analysis is considered as a challenge for the further works.

Table 5. Variation of total cost (SEK) of the inspection scenarios for different combination of C_{τ_1} and C_{τ_2}

Cost values	Inspection scenario			
	6 weeks	12 weeks	24 weeks	48 weeks
$C_{\tau_1} = 100\ and\ C_{\tau_2} = 15{,}000$	3,023,600	2,190,800	1,900,000	2,971,000
$C_{\tau_1} = 1{,}000\ and\ C_{\tau_2} = 15{,}000$	3,509,600	3,054,800	3,318,400	5,489,200
$C_{\tau_1} = 200\ and\ C_{\tau_2} = 5{,}000$	3,077,600	2,286,800	1,957,600	2,350,800
$C_{\tau_1} = 200\ and\ C_{\tau_2} = 50{,}000$	3,077,600	2,286,800	2,407,600	6,400,800

8. Conclusion

This paper proposes using an integrated model to identify a cost-effective track geometry inspection interval. The proposed model contains three sub models: degradation, shock event, and tamping recovery. First, to model track geometry, it applies a degradation Wiener process. The Wiener process is able to capture the uncertainty of track geometry degradation. Second, it uses an exponential distribution to simulate shock event times. If the degradation and shock models are integrated, it is possible to estimate the time to reach the maintenance thresholds. Third, to estimate tamping recovery values, the paper suggests a probabilistic approach, as with this approach, it is possible to predict the long term track geometry condition of each track section. The final integrated model facilitates estimation of total time duration over first and second thresholds, along with the number of inspections and tamping activities in the time horizon of the study. The model allows the comparison of different scenarios for track geometry inspections to find the one with lowest total cost. The case study indicates the simulated track geometry condition using the integrated model follows a pattern similar to the real track geometry condition. The results of the case study clearly show the proposed methodology properly identifies optimal cost-based inspection intervals. The same methodology can be used to identify optimal inspection intervals to suit different objectives.

Acknowledgement

The authors wish to thank Trafikverket and the Luleå Railway Research Centre for the technical support they provided during this project.

References

1. I. Arasteh khouy, "*Cost-effective maintenance of railway track geometry: A shift from safety limits to maintenance limits*,", pp. 1402 2013.
2. I. Soleimanmeigouni and A. Ahmadi, "A Survey on Track Geometry Degradation Modelling," in *Current Trends in Reliability, Availability, Maintainability and Safety*: Springer, 2016, pp. 3-12.
3. S. Tang, X. Guo, Z. Zhou, Z. Zhou and B. Zhang, "Real time remaining useful life prediction based on nonlinear wiener based degradation processes with measurement errors," *J.Cent.South Univ* 2014.
4. X. Si, W. Wang, C. Hu, M. Chen and D. Zhou, "A wiener-process-based degradation model with a recursive filter algorithm for remaining useful life estimation," *Mechanical Systems and Signal Processing*, vol. 35, no. 1, pp. 219-237 2013.
5. W. Kahle and A. Lehmann, "The Wiener process as a degradation model: Modeling and parameter estimation," in *Advances in Degradation Modeling*: Springer, 2010, pp. 127-146.
6. B. Lichtberger, "Track compendium," *Hamburg: Eurailpress, Tetzlaff-Hestra GmbH & CO.KG*, pp. 309-332 2005.
7. A. Lehmann, "Joint modeling of degradation and failure time data," *Journal of Statistical Planning and Inference*, vol. 139, no. 5, pp. 1693-1706 2009.
8. I. Arasteh khouy , P. Larsson-Kråik, A. Nissen, U. Juntti and H. Schunnesson, "Optimisation of track geometry inspection interval," *Proc.Inst.Mech.Eng.Pt.F: J.Rail Rapid Transit*, vol. 228, no. 5, pp. 546-556 2014.